

National Aeronautics and Space Administration



# **Rudder/Speed Brake Actuator Probabilistic Fatigue Life and Reliability Analysis - A Case Study**

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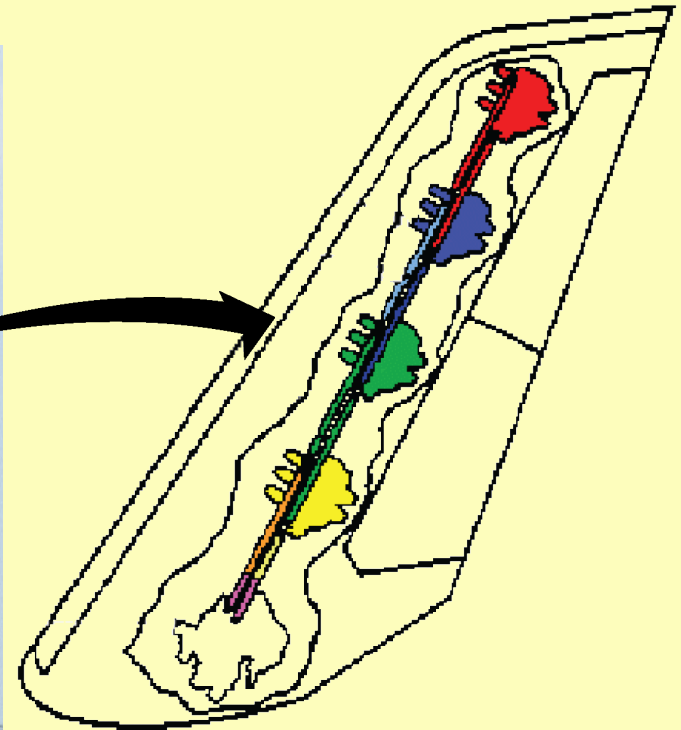
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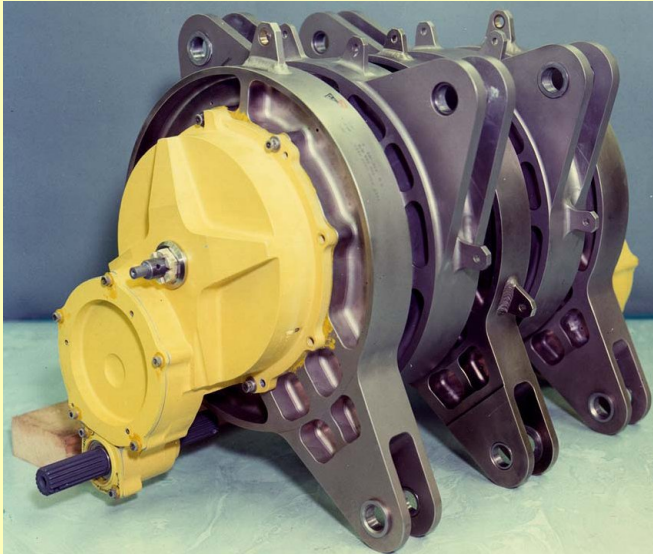


# Rudder/Speed Brake Actuator

**Four actuators, each with two gear trains**

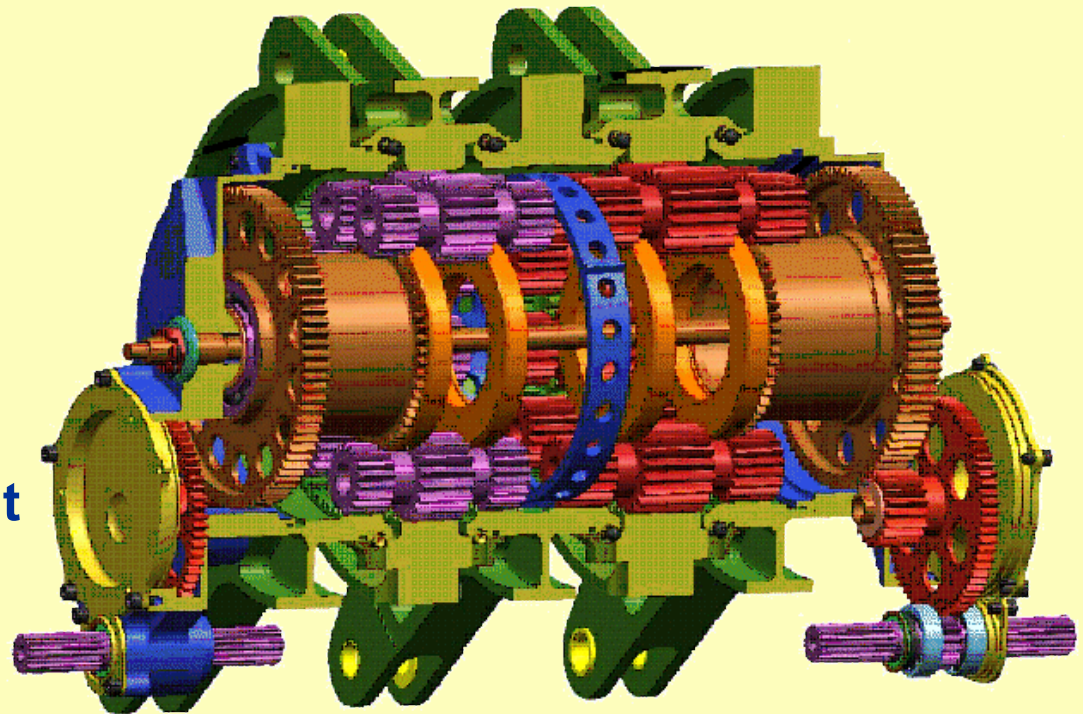


# Rudder/Speed Brake Actuator



Actuator has 2 gear trains —  
each with 17 gears & 10 bearings

- Overall Reduction  
474 : 1
- Two Stage Spur Gear Input  
19.75 : 1
- Differential Planetary  
24 : 1





## Background

- **Space Shuttle designed for 100 missions in 10 years**
- **First Shuttle “Columbia” entered service Apr. 1981**
- **Actuators from Columbia were inspected during vehicle modification after sixth flight in 1984.**
- **No other actuator inspections or maintenance was planned during shuttle program**



## Background

### **Post-Shuttle Columbia crash Feb 2003 . . . .**

- **“Flight leader” Discovery had 30 flights in 19 years**
- **Corrosion & cracks were found on Discovery body flap actuator shaft spline**
- **Wear, damage & red-colored grease were found in rudder/speed brake actuator**



# Issues

- 1. What are the risks from fatigue, wear, lubrication breakdown for 20 or more flights per actuator?**
- 2. What is an acceptable risk for single actuator and four actuators as a system?**



## Method of Analysis

- 1. Load analysis & cycle counts on bearings & gears**
- 2. Probabilistic analysis for life & reliability using Lundberg-Palmgren theory applied to bearings & gears**
- 3. Component life for operating profile in each actuator based on linear damage rule (Palmgren-Langer-Miner rule)**
- 4. System life for single actuator & four actuators based on strict series reliability and Weibull theory**



# Analysis Tasks

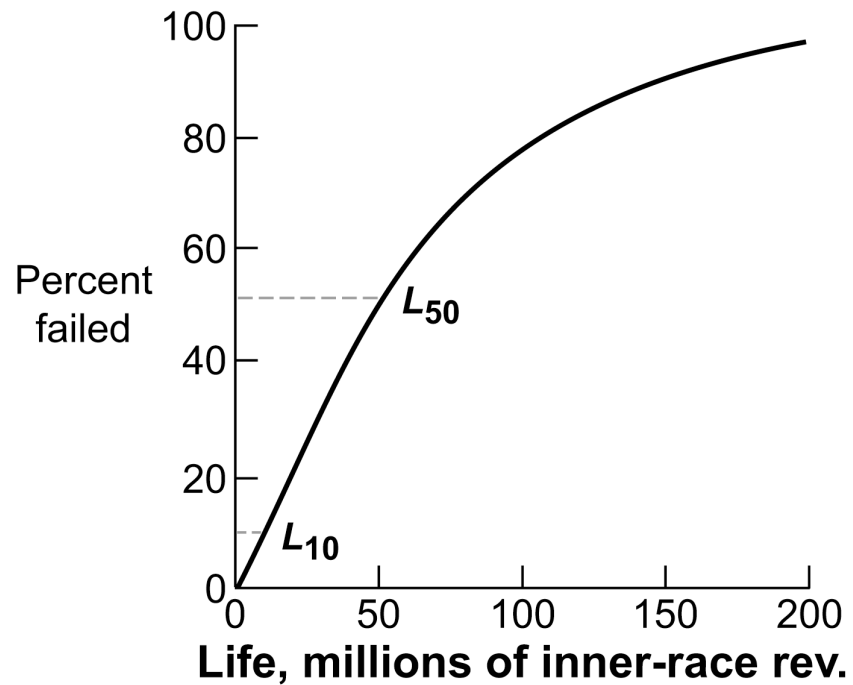
- **Relate Shuttle mission spectrum to cycles of actuator components**
- **Compute gear surface Hertzian fatigue life**
- **Compute gear tooth bending fatigue life**
- **Life analysis for gearbox bearings (supplied by actuator manufacturer)**
- **Combine lives to calculate life of actuator system(s)**



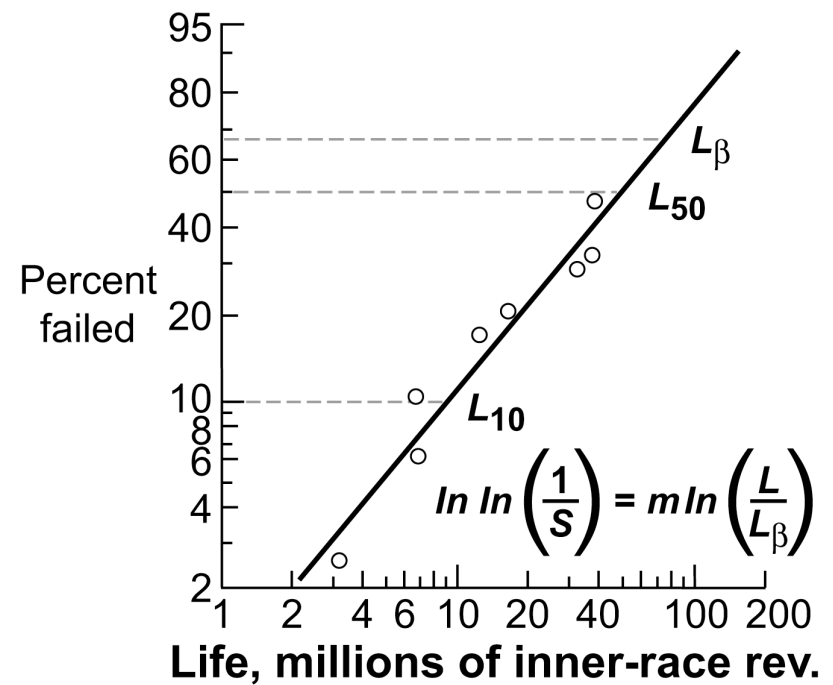


# Weibull Distribution Function

## Linear Coordinates



## Weibull Coordinates





## 2-parameter Weibull Distribution Function

$$\ln \ln \left( \frac{1}{S} \right) = m \ln \left( \frac{L}{L_{\beta}} \right)$$

**where:**

**$S$  = probability of survival or reliability**

**$m$  = Weibull slope (shape factor)**

**$L$  = life @ reliability  $S$**

**$L_{\beta}$  = Characteristic life (life @ 63.2% failure rate)**



## **Lundberg-Palmgren Theory System Life for Multiple Components**

### **Strict Series Reliability**

$$S_S = S_1 \times S_2 \times S_3 \times \dots S_n$$

**where all survival probabilities are at the same time increment**

**Lundberg-Palmgren Theory (1947)  
based on Weibull Distribution Function (1939)**

**Gives:** 
$$\frac{1}{L_S^m} = \left(\frac{1}{L_1}\right)^{m_1} + \left(\frac{1}{L_2}\right)^{m_2} + \left(\frac{1}{L_3}\right)^{m_3} + \dots + \left(\frac{1}{L_n}\right)^{m_n}$$

**where all lives are at the same probability of survival**



## Linear Damage Rule

- **A. Palmgren — 1924**
- **B. Langer — 1937**
- **M. Miner — 1945**

$$\frac{1}{L} = \frac{X_1}{L_1} + \frac{X_2}{L_2} + \frac{X_3}{L_3} + \dots + \frac{X_n}{L_n}$$

$$X_1 + X_2 + X_3 + \dots + X_n = 1$$

**$L \sim$  Life**

**$X \sim$  time fraction**



## System $L_{10}$ Life

$$L_{10} = \left[ \frac{\text{System Dynamic Capacity}}{\text{Equivalent Output Torque}} \right]^p$$

$L_{10}$  = Output cycles @ 90% probability of survival

**From Linear Damage Rule:  
Equivalent Output Torque =**

$$T = \left[ \frac{X_1 T_1^p + X_2 T_2^p + X_3 T_3^p + \dots + X_n T_n^p}{X_1 + X_2 + X_3 + \dots + X_n} \right]^{1/p}$$

$X_n$  = time @ condition  $n$

$T_n$  = torque @ condition  $n$



# Gear Tooth Surface Fatigue Life

$$L_{10tc} = C_{10} W_N^{-4.3} F_e^{3.9} \rho^{-5.0} \ell^{-0.4}$$

**where:**

**$L_{10tc}$  = Contact fatigue 10% life for a single tooth**

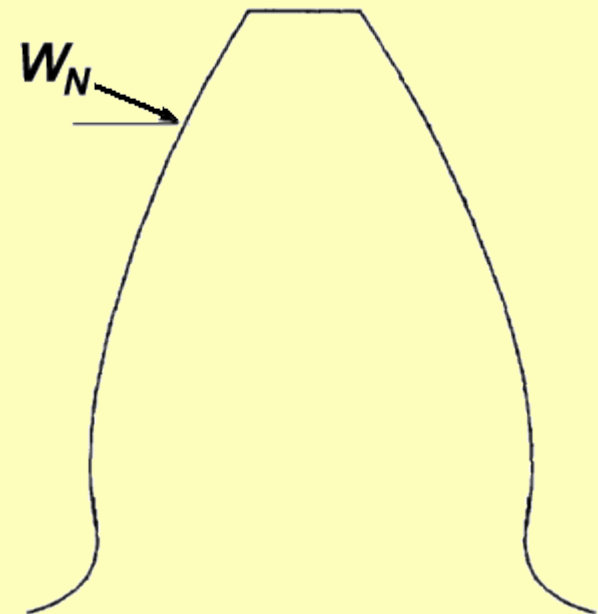
**$C_{10}$  = Gear tooth dynamic load capacity**

**$W_N$  = Normal load on tooth**

**$F_e$  = Effective tooth width**

**$\rho$  = Curvature sum**

**$\ell$  = Length of loaded tooth profile**



# Gear Surface Fatigue Life

$$L_{10tc} = C_{10} W_N^{-4.3} F_e^{3.9} \rho^{-5.0} \square^{-0.4}$$

$$L_{10gc} = L_{10tc} n^{-1/m}$$

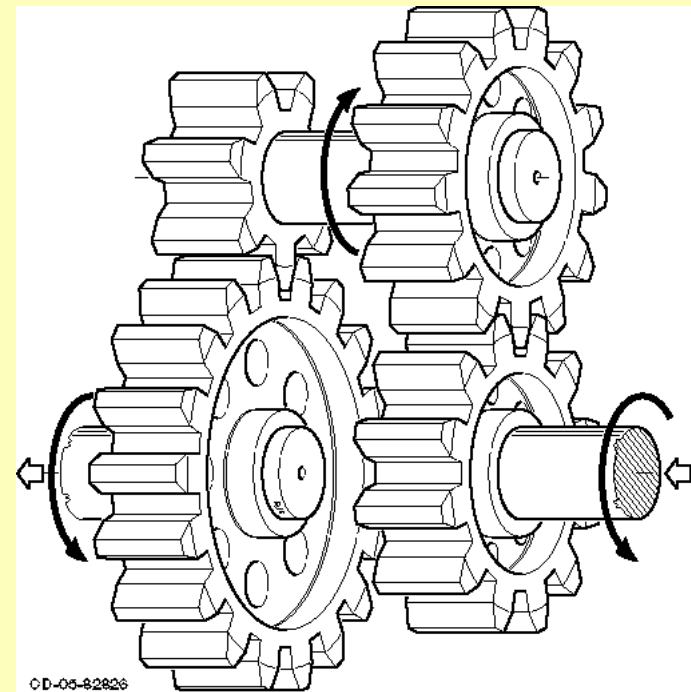
where:

$L_{10tc}$  = Contact fatigue 10% life for a single tooth

$L_{10gc}$  = Surface fatigue 10% life for gear

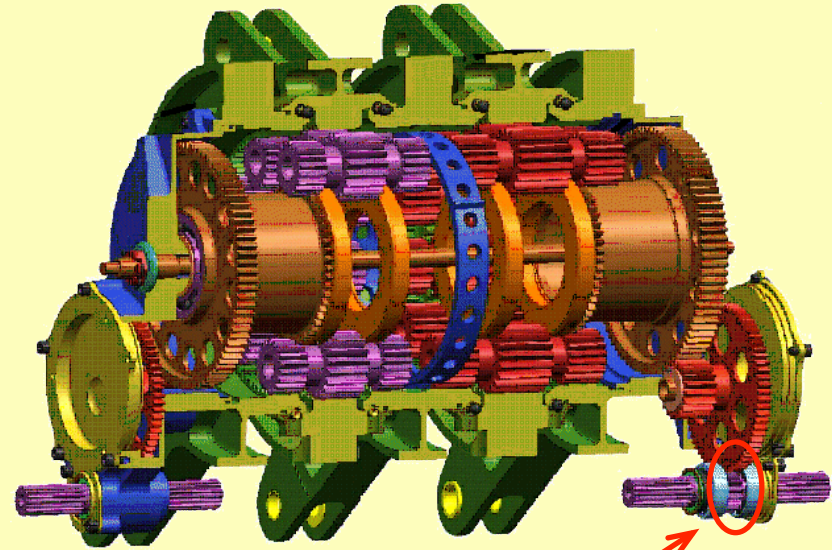
$n$  = Number of teeth

$m$  = Weibull slope = 2.5



# Gear Surface Fatigue Life

$$L_{10gc} = L_{10tc} n^{-1/m}$$



**Lowest-life gear: 16 tooth input pinion**

**Max Hertz stress 1818 MPa (263.7 ksi)**

**Pinion  $L_{10} = 1.807 * 10^{12}$  cycles**

**based on weighted output torque 29.8 Nm (263.7 in-lb)**

**Gear system life (contact):**

$$\frac{1}{L_{Sc}^m} = \left( \frac{1}{L_{1c}} \right)^{m_1} + \left( \frac{1}{L_{2c}} \right)^{m_2} + \left( \frac{1}{L_{3c}} \right)^{m_3} + \dots + \left( \frac{1}{L_{nc}} \right)^{m_n}$$





# Gear Tooth Bending Fatigue Life

## AGMA Bending Stress Number

$$s_t = W_t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

where:

$s_t$	= Bending stress number, lb/in <sup>2</sup>
$W_t$	= Tangential load, lb
$K_o$	= Overload factor
$K_v$	= Dynamic factor
$K_s$	= Size factor
$P_d$	= Diametral pitch, in <sup>-1</sup>
$F$	= Face width, in
$K_m$	= Load distribution factor
$K_B$	= Rim thickness factor
$J$	= Geometry factor



# Gear Tooth Bending Fatigue Life

$$L_{10gb} = \left( \frac{S}{s_t} \right)^p$$

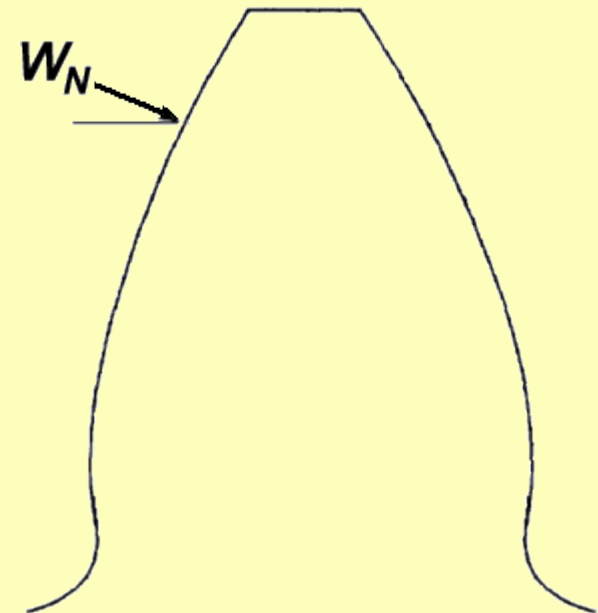
**where:**

**$L_{10gb}$  = Bending stress life for a single tooth**

**$S$  = Gear tooth strength (database)**

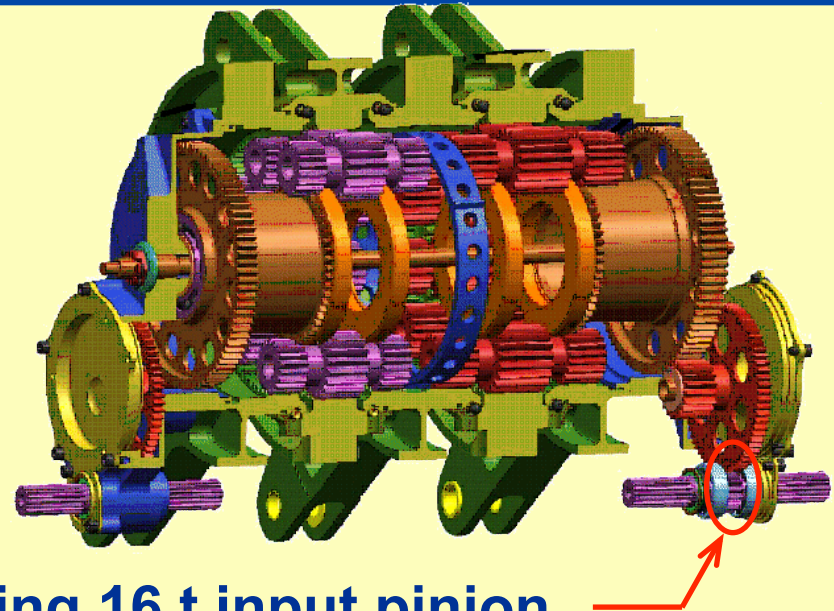
**$s_t$  = Bending stress**

**$p$  = Stress-life exponent**



# Gear Tooth Bending Fatigue Life

$$L_{10gb} = \left( \frac{S}{S_t} \right)^p$$



**Lowest-life gear for tooth bending 16 t input pinion**

**Bending stress 1070 MPa (155 ksi),**

**$L_{10b}$  life =  $13 \times 10^{12}$  cycles**

**Gear system life (bending):**

$$\frac{1}{L_{Sb}^m} = \left( \frac{1}{L_{1b}} \right)^{m_1} + \left( \frac{1}{L_{2b}} \right)^{m_2} + \left( \frac{1}{L_{3b}} \right)^{m_3} + \dots + \left( \frac{1}{L_{nb}} \right)^{m_n}$$



# Gear Tooth Load Cycles (for 1 tooth @ Output)

<b>Gear</b>	<b>No Teeth</b>	<b>Teeth – load cycles</b>	<b>Mesh Cycles</b>
Output Ring	81	9 – 1	9
Output Planet	18	9 – 1	9
Fixed Ring	90	9 – 1	9
Fixed Planets	18	9 – 1	9
Sun	54	36-1	36
Sun-Planets	18	36-1	36
Output Spur	87	26-1	26
Intermed. Output	19	12-1 + 7-2	26
Intermed. Input	69	44-1+ 25-2	94
Input Spur	16	2-5 + 14-6	94

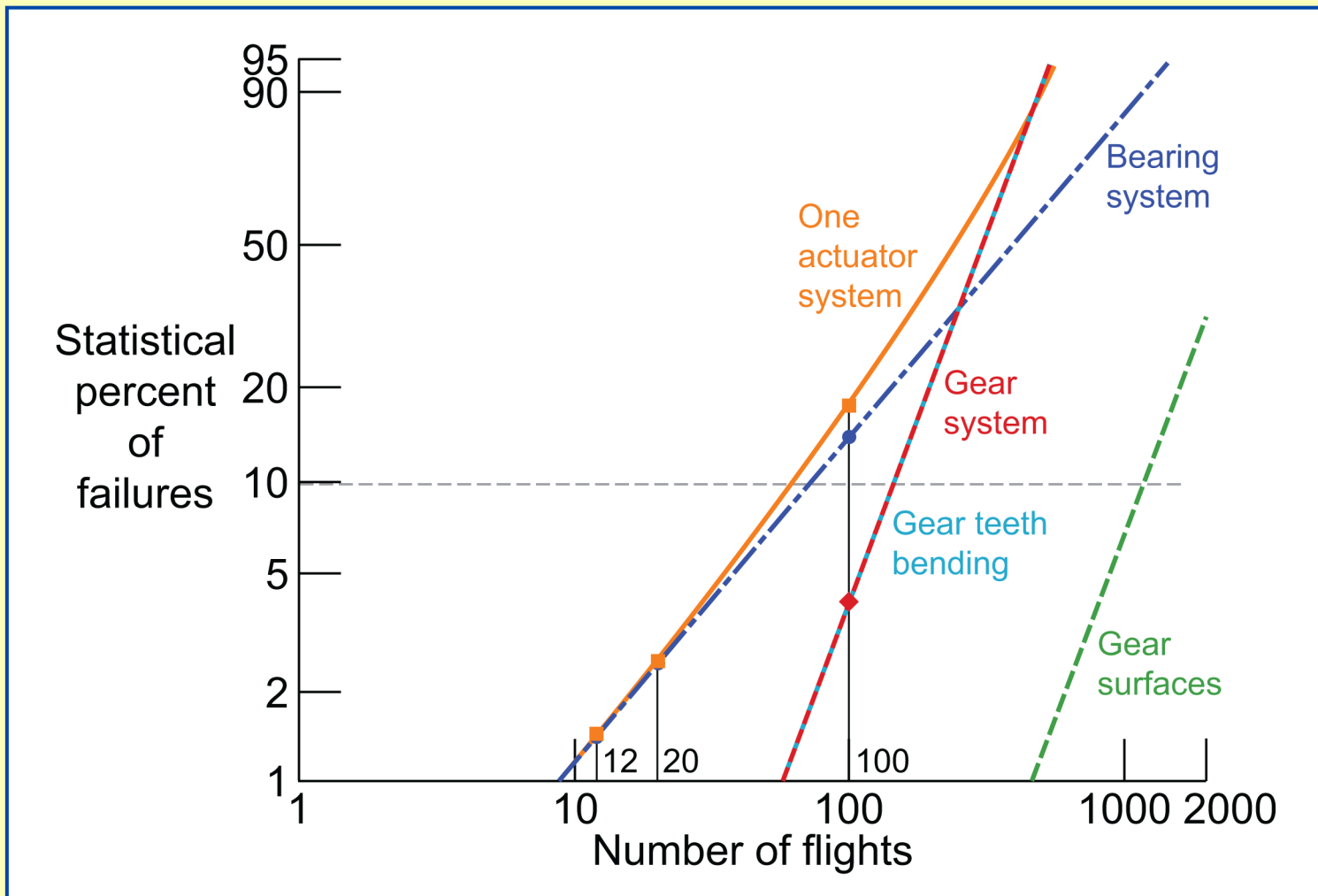


## Load Spectrum (for 100 flights)

Event	Moment N-m (in-lb)	Duration (Minutes)	Output gear teeth loading cycles
Ferry	560 (5,000)	420	32,392,514
Ferry	24,000 (212,000)	0.25	19,281
Ascent	9,000 (80,000)	0.2	15,425
Ascent	5,200 (46,000)	1.7	131,113
Ascent	11,000 (98,000)	0.1	7,713
Descent	16,000 (140,000)	33.0	2,545,126
Descent	27,000 (240,000)	1.0	77,125

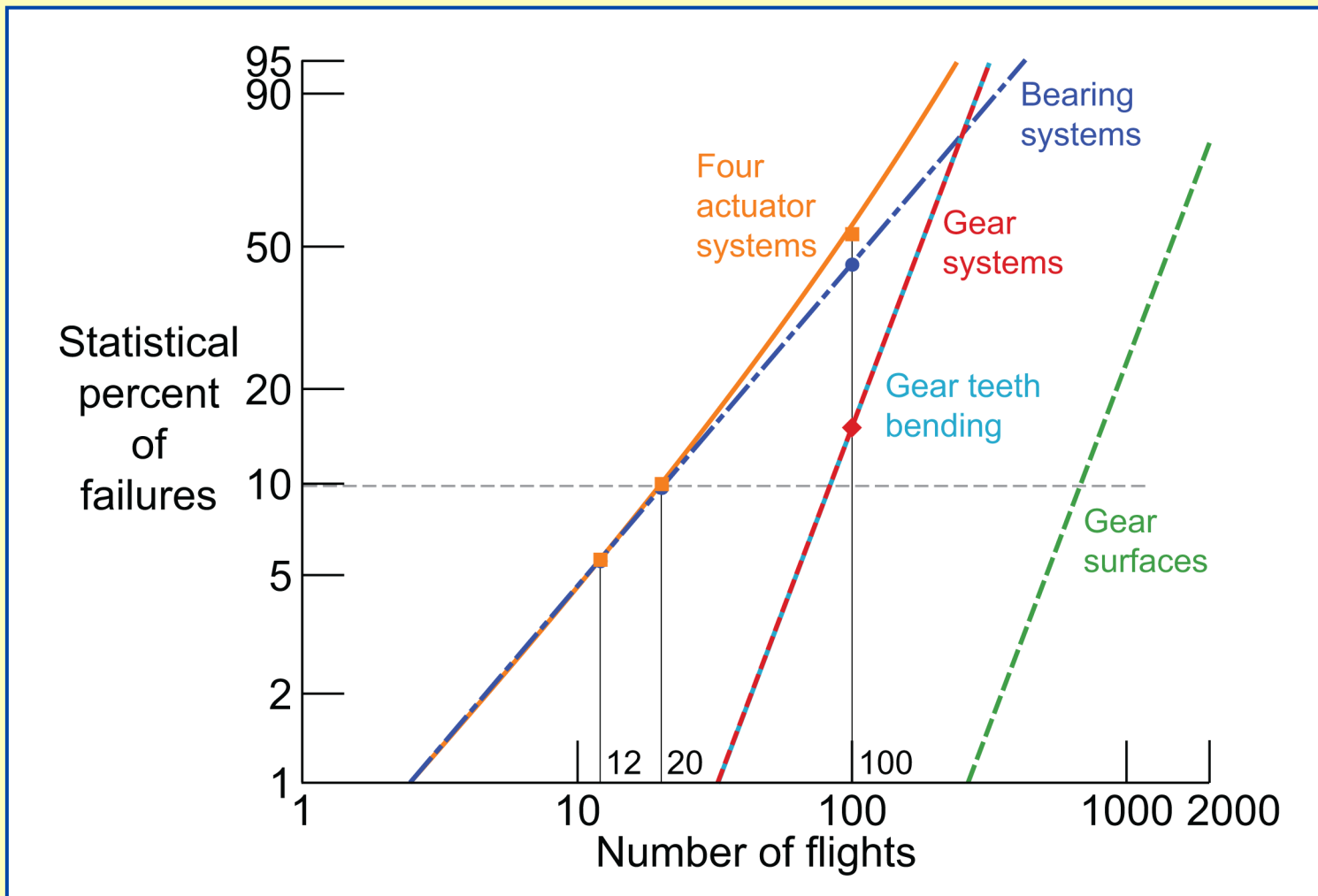


# Life vs. Reliability – 1 Actuator



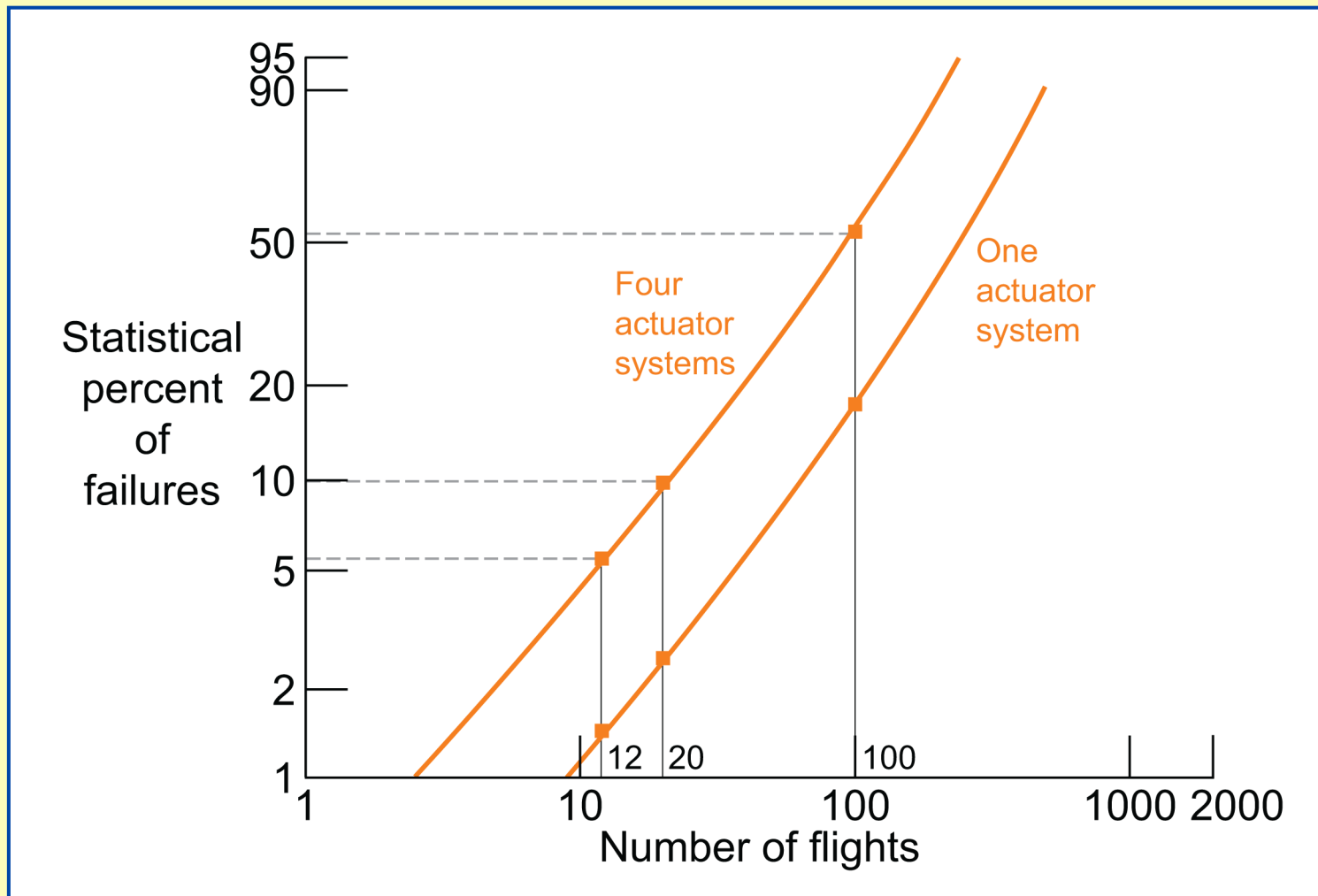


# Life vs. Reliability – 4 Actuators





# Comparison of Life & Reliability for 1 & 4 Actuators







## Predicted Lives of Rudder/Speed Brake Actuator Bearings

Bearing No.	1a	1b	2	3a	3b	4	6a	6b
Bearing type	roller	roller	roller	roller	roller	ball	ball	ball
$L_{10B}$ life (hrs)	647,000	647,000	15,823	4,875	4,875	54,973	1,509	1,089

$L_{10B}$  life data provided for bearings by actuator manufacturer

No bearing failure expected before 5.3% of min.  $L_{10B}$  life:

$$L_{10B} = 0.053(1089 \text{ hrs}) = 58 \text{ hrs or 7 flights}$$



# Rudder/Speed Brake Actuator Probability of Survival

Number of flights	Number of actuators	Gear Reliability, percent			Actuator Bearing Reliability, percent	Total System Reliability, percent
		Tooth Bending Fatigue	Tooth Surface Fatigue	Combined Bending & Surface		
100	1	95.9	99.98	95.9	86.0	82.4
100	4	84.71	99.91	84.7	54.6	46.2
20	1	99.93	99.999+	99.93	97.5	97.4
20	4	99.7	99.998	99.7	90.3	90.1
12	1	99.98	99.999+	99.98	98.6	98.6
12	4	99.92	99.998	99.92	94.4	94.3



## Rudder/Speed Brake Actuator Probability of Survival

Number of Actuators	Number of Flights			
	7	12	20	100
	Probability of Survival (failure), %			
1	100	98.6	97.4	82
4	100	94	90	46
Bearing System	100	99.8	97.5	86
Weakest Bearing	100	99.3	98.8	93

90% reliability (10% failure) for 20 flights unacceptable

New goal 12 flights, 94% reliability (6% failure)

No failures predicted for up to 7 flights



## Summary of Results

- **Rudder/Speed Brake Actuators limited to 12 flights**
- **Reliability for 12 flights: 98.6% for one actuator, 94% for four actuators**
- **No failures expected for up to 7 flights**
- **Four actuator system reliability: 90% for 20 flights and 46% for 100 flights**
- **Life & reliability of actuator system dominated by bearings**



## Epilogue

- **Shuttle Discovery returned to flight, July 2005**
- **Shuttle Atlantis concluded program, July 2011**
- **Space Shuttle Program had a total of 135 missions from 1981 — 2011**
- **At retirement, flight leader, Discovery had 39 missions in 27 years (9 after return to flight)**
- **22 missions for 3 remaining shuttles from return to flight until end of program**